Multistage decomposition algorithm for blind source separation*

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Abstract A new algorithm for blind source separation is proposed, which only extracts the single independent component at each stage. The single independent component is acquired by an iterative algorithm for searching for the optimal solution of the defined cost function. Moreover, all the independent components are obtained by systematic multistage decomposition and multistage reconstruction. When there is spatially colored noise, the performance of this algorithm is advantageous over jointly approximated diagonalization of eigenmatrices (JADE). Simulated results show that if the number of source signals is more than 25, its computational complexity is lower than that of JADE.

Keywords: **blind source separation, multistage decomposition, multistage reconstruction, criterion, iteration algorithm.**

In recent two decades, many blind source separation (BSS) algorithms have been proposed. The representative off-line approaches are the matrix-pencil algorithms based on two correlation or higher-order cumulant matrices (called eigen matrices), which include the algorithm for multiple unknown signal extraction (AMUSE)[1,2] and Blind beamforming[3]. They adopted basically the approach of estimation of signal parameters via rotational invariance techniques (ESPRIT)[4]. The matrix-pencil algorithms based on two eigen matrices usually get all the independent components fast. Unfortunately, their performance is inferior to the JADE and the eigenvalues often degenerate[5]. In order to improve algorithms and to prevent eigenvalue-spectral from degeneracy, Cardoso et al. proposed JADE technique[5,6]. It starts with prewhitening, which transforms the response matrix to some unknown unitary matrix; then to estimate the unitary matrix by "joint diagonalization" of the whole set of the fourth-order cumulant or correlation matrices of the whitened process. Like the Jacobi techniques in matrix eigenvalue decomposition, the JADE is also excellent. In Ref. [7], the JADE criterion based on the notion of the "contrast function" is shown as the least squares solution to joint diagonalization problem. However, since the prewhitening is dependent on the assumption in which noises are independently identical distribution (i. i. d.), when noises are not i.i.d. or the sampling number is limited, "whitening" scheme is approximately implemented and will increase the additional error[1,5]. Although the improved least squares criterions[7] do not need assume that noise is i.i.d., to find the optimal solution is difficult since they are some highly nonlinear function.

Here, a new jointly approximated diagonalization of a set of eigen (correlation) matrices is proposed. Even though prewhitening is not implemented, it can also get all the independent components. Based on biorthogonality between the left and right eigenvectors and the structural information in a set of eigen (correlation) matrices, we present a kind of cost functions for finding the single independent component. An optimal solution to the cost functions can efficiently be obtained by an iteration algorithm. Simulated results show that the iteration algorithm converges to a fixed point within 10 rounds of iteration on average. By systematic multistage decomposition and multistage reconstruction one can get all the independent components[8]. When noise is not i.i.d., the proposed algorithm is advantageous over the JADE techniques[5]. And when the number of independent sources is more than 25, the computational complexity of this algorithm is lower than that of the JADE.

1 Blind source separation problems and indeterminacy

1.1 Signal model

Consider a linear array composed of m sensors.

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Assume that n narrow-band sources centered around a known frequency impinge on the array from distinct directions. The m-dimensional data vector received by the array is expressed as

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \quad (1)$$

where $\boldsymbol{A} \in \mathbb{C}^{m\times n}$ is the array response matrix, $\boldsymbol{s}(t) \in \mathbb{C}^{n\times 1}$ the source signal vector, $\boldsymbol{x}(t) \in \mathbb{C}^{m\times 1}$ the array output vector, and $\boldsymbol{n}(t) \in \mathbb{C}^{m\times 1}$ the noise vector. Given T samples of array output vector $\{\boldsymbol{x}(t_i)\}_{i=1}^{T}$, the blind source separation problem is to estimate the array response matrix $\boldsymbol{A}$ from the sampled data $\{\boldsymbol{x}(t_i)\}_{i=1}^{T}$ and further retrieve the source signals $\{\boldsymbol{s}(t_i)\}_{i=1}^{T}$.

To solve this problem, we make the following assumptions.

(i) The unknown response matrix $\boldsymbol{A}$ is full column rank.

(ii) The source signal vector $s(t)$ is a stationary multivariate process, i.e.

$$E\{\boldsymbol{s}(t+\tau)\boldsymbol{s}^{\mathrm{H}}(t)\} = \mathrm{diag}[\rho_1(\tau), \cdots, \rho_n(\tau)], \quad (2)$$

where superscript H denotes the conjugate transpose, and diag[·] indicates the diagonal matrix.

(iii) At most, one of the source signals may be Gaussian white noise.

(iv) The additive noise vector $\boldsymbol{n}(t)$ is zeros-mean Gaussian noise independent of the source signals, i.e.

$$E\{\boldsymbol{n}(t+\tau)\boldsymbol{n}^{\mathrm{H}}(t)\} = \delta(\tau)\mathrm{diag}[r_1^2, \cdots, r_m^2], \quad (3)$$

where $\delta(\tau)$ is the Kronecker function.

Under the above assumptions, the correlation matrices of the array output vector have the following structure:

$$\begin{aligned}\boldsymbol{R}_x(0) &= E\{\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)\} \\ &= \boldsymbol{A}\,\mathrm{diag}[\rho_1(0), \cdots, \rho_n(0)]\boldsymbol{A}^{\mathrm{H}} \\ &\quad + \mathrm{diag}[r_1^2, \cdots, r_m^2], \quad (4)\\ \boldsymbol{R}_x(\tau) &= E\{\boldsymbol{x}(t+\tau)\boldsymbol{x}^{\mathrm{H}}(t)\} \\ &= \boldsymbol{A}\,\mathrm{diag}[\rho_1(\tau), \cdots, \rho_n(\tau)]\boldsymbol{A}^{\mathrm{H}}. \quad (5)\end{aligned}$$

In the following sections, we show how to estimate the response matrix and the signals $\{\boldsymbol{s}(t_i)\}_{i=1}^{T}$ from the measured data $\{\boldsymbol{x}(t_i)\}_{i=1}^{T}$, without a priori knowledge of the array manifold.

1.2 Indeterminacy of blind identification

In the blind context, fully identifying for the mixture matrix $\boldsymbol{A}$ is impossible because the exchange of a fixed scalar factor between a given source signal and the corresponding column of $\boldsymbol{A}$ does not affect the measured data[9]. Even though there is an undetermined complex constant in each column of $\boldsymbol{A}$, the directions of the columns of $\boldsymbol{A}$ can be uniquely determined. Without loss of generality, let all the columns be unit vectors.

True indeterminacy arises from degeneration of eigenvalues[5,6]. In order to improve estimation, we should study the simultaneous diagonalization of a set of eigen matrices $\{\boldsymbol{R}_x(\tau_i)\}_{i=0}^{K-1}$.

2 Approach for extracting single independent component

2.1 Biorthogonality of matrices

For matrix $\boldsymbol{A} = [\boldsymbol{a}_1, \cdots, \boldsymbol{a}_n] \in \mathbb{C}^{m\times n}$, its Moore-Penrose pseudo inverse is expressed by $\boldsymbol{A}^{+} = [\boldsymbol{a}_1^{+}, \cdots, \boldsymbol{a}_n^{+}]^{\mathrm{H}} \in \mathbb{C}^{m\times n}$. Obviously, the following biorthogonality should hold

$$(\boldsymbol{a}_i^{+})^{\mathrm{H}}\boldsymbol{a}_j = \boldsymbol{a}_j^{\mathrm{H}}\boldsymbol{a}_i^{+} = \delta(i-j), \quad (6)$$

where $\boldsymbol{a}_j^{+}$ $(j = 1, \cdots, n)$ are called the adjoint vectors of $\boldsymbol{a}_j$ $(j = 1, \cdots, n)$.

2.2 Dimension-reduced processing

By dimension reducing, one can obtain the $n \times n$ squares mixture matrix from the known $m \times n$ $(m > n)$ nonsquares mixture matrix. The ideal dimension-reduced matrix $\boldsymbol{T} \in \mathbb{C}^{m\times n}$ should satisfy the condition $\mathrm{span}(\boldsymbol{T}) = \mathrm{span}(\boldsymbol{A})$. Moreover, for simplicity, $\boldsymbol{T}$ should best be selected as a unitary matrix. In Refs. [1, 5], $\boldsymbol{T}$ is specially chosen as the whitening matrix, which is an approximate dimension-reduced matrix gotten only by the eigen value decomposition of $\boldsymbol{R}_x(0)$. When there is spatially colored noise, its precision is low, while that of the dimension-reduced matrix derived from $\{\boldsymbol{R}_x(\tau_i)\}_{i=1}^{K-1}$ is high. Let matrix $\boldsymbol{C} = \sum_{i=1}^{K-1}\boldsymbol{R}_x^{\mathrm{H}}(\tau_i)\boldsymbol{R}_x(\tau_i)$, then the singular value decomposition (SVD) of $\boldsymbol{C}$ is represented as $\boldsymbol{C} = \boldsymbol{U}\boldsymbol{D}\boldsymbol{U}^{\mathrm{H}}$, where $\boldsymbol{U} = [\boldsymbol{u}_1, \cdots, \boldsymbol{u}_m] \in \mathbb{C}^{m\times m}$. The number of source signals is determined by the number of the dominant singular values of $\boldsymbol{C}$, then the dimension-reduced matrix is given by $\boldsymbol{T} =$

$[\boldsymbol{u}_1, \cdots, \boldsymbol{u}_n]$.

After dimension reducing, the array still obeys a linear model, i.e.

$$\boldsymbol{z}(t) \overset{\text{def}}{=\!=} \boldsymbol{T}^{\mathrm{H}}\boldsymbol{x}(t) = \boldsymbol{T}^{\mathrm{H}}\boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{T}^{\mathrm{H}}\boldsymbol{n}(t) = \boldsymbol{B}_0\boldsymbol{s}(t) + \boldsymbol{T}^{\mathrm{H}}\boldsymbol{n}(t), \quad (7)$$

where $\boldsymbol{B}_0 = \boldsymbol{T}^{\mathrm{H}}\boldsymbol{A} \in \mathbb{C}^{n\times n}$ is a full-rank squares matrix. Notice that in Eq. (7) the noise is suppressed, while the signals are maintained.

2.3 Criterion

Define the spatially dimension-reduced correlation matrices as

$$\begin{aligned}\boldsymbol{R}_0(0) &= E\{\boldsymbol{z}(t)\boldsymbol{z}^{\mathrm{H}}(t)\} \\ &= \boldsymbol{B}_0\mathrm{diag}[\rho_1(0), \cdots, \rho_n(0)]\boldsymbol{B}_0^{\mathrm{H}} \\ &\quad + \boldsymbol{T}^{\mathrm{H}}\mathrm{diag}[r_1^2, \cdots, r_m^2]\boldsymbol{T}, \quad (8)\\ \boldsymbol{R}_0(\tau) &= E\{\boldsymbol{z}(t+\tau)\boldsymbol{z}^{\mathrm{H}}(t)\} \\ &= \boldsymbol{B}_0\mathrm{diag}[\rho_1(\tau), \cdots, \rho_n(\tau)]\boldsymbol{B}_0^{\mathrm{H}} \\ &= \sum_{i=1}^{n}\rho_i(\tau)\boldsymbol{b}_{0i}\boldsymbol{b}_{0i}^{\mathrm{H}}, \quad \tau \neq 0. \quad (9)\end{aligned}$$

Let $\boldsymbol{b}_{0i}^{+}$ be the adjoint vector of $\boldsymbol{b}_{0i}$. It follows from (6) that

$$\boldsymbol{R}_0(\tau)\boldsymbol{b}_{0i}^{+} = \rho_i(\tau)\boldsymbol{b}_{0i}, \quad \boldsymbol{R}_0^{\mathrm{H}}(\tau)\boldsymbol{b}_{0i}^{+} = \rho_i^{*}(\tau)\boldsymbol{b}_{0i}. \quad (10)$$

Thus, we have the criterion for extracting the single independent component as follows

$$\min J_2(\boldsymbol{w}, c_1, \cdots, c_{K-1}) = \sum_{k=1}^{K-1} \| \boldsymbol{R}_0(\tau_k)\boldsymbol{w} - c_k\boldsymbol{R}_0(0)\boldsymbol{w} \|^2, \quad (11)$$

where $\tau_k \neq 0 (k \geqslant 1)$. If $\boldsymbol{w}$ or $c_0, \cdots, c_{K-1}$ are fixed, this criterion is an aquadratic function. While, the least square criterion defined by Wax and Anu[7] is of the 4th power, even though the elements of the diagonal matrix are fixed.

2.4 Tracking single independent component

The iteration for finding the weight vector starts with a randomly generated initial value of $\boldsymbol{w}$. In order to get the better estimate, the first principal component of $\boldsymbol{R}_0(0)$ can be chosen as the initial value of $\boldsymbol{w}$. At the kth step, a set of parameters is first computed such that $\| \boldsymbol{R}_0(\tau_k)\boldsymbol{w} - c_k\boldsymbol{R}_0(0)\boldsymbol{w} \|^2$ is minimized. Thus, for $k = 1, 2, \cdots$, the iterating steps are as follows.

(i) Compute

$$c_i(k) = [\boldsymbol{R}_0(0)\boldsymbol{w}(k-1)]^{\mathrm{H}} \cdot [\boldsymbol{R}_0(\tau_i)\boldsymbol{w}(k-1)] / \| \boldsymbol{R}_0(0)\boldsymbol{w}(k-1) \|^2; \quad (12)$$

(ii) find unit-norm weight vector $\boldsymbol{w}(k)$ to make

$$\boldsymbol{w}^{\mathrm{H}}(k)\Big\{\sum_{i=1}^{K-1}[\boldsymbol{R}_0(\tau_i) - c_i(k)\boldsymbol{R}_0(0)]^{\mathrm{H}} \cdot [\boldsymbol{R}_0(\tau_i) - c_i(k)\boldsymbol{R}_0(0)]\Big\}\boldsymbol{w}(k) \quad (13)$$

be minimized, i.e. let $\boldsymbol{w}(k)$ be equal to the eigenvector associated with the smallest eigenvalue of the following matrix

$$\boldsymbol{C}(k) = \sum_{i=1}^{K-1}[\boldsymbol{R}_0(\tau_i) - c_i(k)\boldsymbol{R}_0(0)]^{\mathrm{H}} \cdot [\boldsymbol{R}_0(\tau_i) - c_i(k)\boldsymbol{R}_0(0)]; \quad (14)$$

(iii) repeat the above two steps till $\| \boldsymbol{w}(k) - \boldsymbol{w}(k-1) \| < \varepsilon$ (in our calculations $\varepsilon = 10^{-12}$), and take $\boldsymbol{b}_0^{+} = \boldsymbol{w}(k)$.

On computing, at each step of (i) and (ii), only a least squares problem is accurately solved. It must be mentioned that the problem of the convergence and convergent speed of the above iteration is still unsolved. Simulated results show that the iteration algorithm converges to a fixed point within about 10 rounds of iteration on average.

2.5 Determination of single independent component

Once $\boldsymbol{b}_0^{+}$ is obtained, it can be known from the biorthogonality that $\boldsymbol{b}_0$ is equal to the eigenvector associated with the largest eigenvalue of matrix $\boldsymbol{F}_0 = \sum_{i=0}^{K-1}[\boldsymbol{R}_0(\tau_i)\boldsymbol{b}_0^{+}][\boldsymbol{R}_0(\tau_i)\boldsymbol{b}_0^{+}]^{\mathrm{H}}$.

3 Multiple independent component extraction by multistage decomposition

3.1 First stage decomposition

Once a column of $\boldsymbol{b}_0$ and its adjoint vector $\boldsymbol{b}_0^{+}$ are gotten, the component $\rho(\tau)\boldsymbol{b}_0\boldsymbol{b}_0^{\mathrm{H}}$ in $\boldsymbol{R}_0(\tau)$ can be removed by $\tilde{\boldsymbol{R}}_0(\tau) = \boldsymbol{R}_0(\tau) - [\boldsymbol{R}_0(\tau)\boldsymbol{b}_0^{+}]\boldsymbol{b}_0^{\mathrm{H}}$, where $\boldsymbol{b}_0^{\mathrm{H}}\boldsymbol{b}_0^{+} = 1$. Note that $\tilde{\boldsymbol{R}}_0(\tau)$ is full rank and its rank is $n-1$. In order to improve the computational efficiency for extracting the next independent component, $\tilde{\boldsymbol{R}}_0(\tau)$ should be reduced to $(n-1)\times(n-1)$ matrix, which can not only prevent the weight vector from converging to the already obtained independent component $\boldsymbol{b}_0$ but also improve the computational efficiency.

From biorthogonality, we have $\tilde{\boldsymbol{R}}_0(\tau)\boldsymbol{b}_0^{+} = 0$, which means that $\mathrm{span}[\tilde{\boldsymbol{R}}_0(\tau)] \perp \boldsymbol{b}_0^{+}$. And, from

the Householder transformation matrix associated with vector $\boldsymbol{b}_0^+$, we can get the following dimension-reduced matrix:

$$\boldsymbol{T}_1 = [\boldsymbol{0} \quad \boldsymbol{I}_{n-1}] - \tilde{\boldsymbol{b}}_0^+(\tilde{\boldsymbol{b}}_0^+ + \boldsymbol{e}_1)^{\mathrm{H}}/(1 + b_{01}^+) \in \mathbb{C}^{(n-1)\times n} \tag{15}$$

where $\boldsymbol{I}_{n-1}$ denotes the $(n-1)\times(n-1)$ identity matrix, $\boldsymbol{0}$ is $(n-1)$ zero vector, $\boldsymbol{e}_1 = [1, 0, \cdots, 0]^{\mathrm{T}} \in \mathbb{R}^{n\times 1}$. If $\boldsymbol{b}_0^+ = [b_{01}^+, \cdots, b_{0n}^+]^{\mathrm{T}}$, then $\tilde{\boldsymbol{b}}_0^+ = [b_{01}^+, \cdots, b_{0n-1}^+]^{\mathrm{T}}$. It is easy to show that $\boldsymbol{T}_1\boldsymbol{b}_0^+ = 0$, thus $\boldsymbol{T}_1$ lies in the orthogonal subspace to $\boldsymbol{b}_0^+$. Even though $\tilde{\boldsymbol{R}}_0(\tau)$ includes noise, we have also span$[\boldsymbol{T}_1] = $ span$[\tilde{\boldsymbol{R}}_0(\tau)]$. This shows that $\boldsymbol{T}_1$ is an ideal and efficient dimension-reduced matrix. Hence, the new dimension-reduced correlation matrix is given by

$$\boldsymbol{R}_1(\tau) = \boldsymbol{T}_1\tilde{\boldsymbol{R}}_0(\tau)\boldsymbol{T}_1^{\mathrm{H}} \in C^{(n-1)\times(n-1)}. \tag{16}$$

The first stage of decomposition is reached.

3.2 Multistage decomposition

The new $(n-1)$-dimensional independent component can be estimated from $\boldsymbol{R}_1(\tau)$. Except that the dimension of independent component is reduced from n to $(n-1)$, the process for finding new independent component is the same as the above decomposition. Thus, from $\boldsymbol{R}_1(\tau)$, an independent component $\boldsymbol{b}_1$ and its adjoint vector $\boldsymbol{b}_1^+$ can be derived by the method given in Sec. 2.4. Decomposition process is finished till stage $(n-2)$.

Given q $(1 \leqslant q \leqslant n-2)$, $\boldsymbol{R}_0(\tau_i)$ $(i = 0, \cdots, K-1)$ in (12), (13) and (14) are replaced by $\boldsymbol{R}_q(\tau_i)$ $(i = 0, \cdots, K-1)$. Notice that the weight vector is $(n-q+1)$-dimensional, $\boldsymbol{b}_q$ and $\boldsymbol{b}_q^+$ can be gotten by a similar approach. Once the weight vector converges to a fixed vector, $\boldsymbol{b}_q^+ = \boldsymbol{w}(k)$ and $\boldsymbol{b}_q$ is obtained by making

$$\boldsymbol{b}_q^{\mathrm{H}}\Big\{\sum_{i=0}^{K-1}[\boldsymbol{R}_q(\tau_i)\boldsymbol{b}_q^+][\boldsymbol{R}_q(\tau_i)\boldsymbol{b}_q^+]^{\mathrm{H}}\Big\}\boldsymbol{b}_q$$

be maximized.

The dimension-reduced matrix should satisfy $\boldsymbol{T}_{q+1} = \mathrm{null}(\boldsymbol{b}_q^+)$ and may be computed using the method similar to computing $\boldsymbol{T}_1$. A series of the dimension-reduced eigen matrices are recursively computed by

$$\boldsymbol{R}_{(q+1)}(\tau) = \boldsymbol{T}_q[\boldsymbol{R}_q(\tau) - (\boldsymbol{R}_q(\tau)\boldsymbol{b}_q^+)\boldsymbol{b}_q^{\mathrm{H}}]\boldsymbol{T}_q^{\mathrm{H}}. \tag{17}$$

When $q = n-2$, $\boldsymbol{R}_{(n-2)}(\tau)$ are the 2×2 eigen matrices. Given $\boldsymbol{b}_{(n-2)}$ and $\boldsymbol{b}_{(n-2)}^+$ involved in $\mathbb{C}^2$, the last independent component situated in $\mathbb{C}^2$ can uniquely be achieved by the following biorthogonality relation $\boldsymbol{b}_{(n-2)}^+ \perp \boldsymbol{b}_{(n-1)}$ and $\boldsymbol{b}_{(n-2)} \perp \boldsymbol{b}_{(n-1)}^+$. This means that the last two independent components can simultaneously be gotten by one step.

3.3 Multistage reconstruction

Once the dimension-reduced independent components are obtained, the full-dimension independent components can be reconstructed by the following process:

Let $\boldsymbol{B}_{(n-2)} = [\boldsymbol{b}_{(n-2)} \vdots \boldsymbol{b}_{(n-1)}] \in \mathbb{C}^{2\times 2}$, then $\boldsymbol{B}_{(q-1)} = [\boldsymbol{b}_{(q-1)} \vdots \boldsymbol{T}_q^{\mathrm{H}}\boldsymbol{B}_q] \in \mathbb{C}^{(n-q+1)\times(n-q+1)}, \cdots,$ $\boldsymbol{B}_0 = [\boldsymbol{b}_0 \vdots \boldsymbol{T}_1^{\mathrm{H}}\boldsymbol{B}_1] \in \mathbb{C}^{n\times n}$.

Since $\boldsymbol{T}_q$ is the last $(n-q)$ rows of $(n-q+1)\times(n-q+1)$ Householder matrix, the matrix product $\boldsymbol{T}_q^{\mathrm{H}}\boldsymbol{B}_q$ is achieved by multiplication of matrix with vector. Therefore, the reconstructing process is computationally efficient.

Finally, the mixture matrix is computed by $\boldsymbol{A} = \boldsymbol{T}\boldsymbol{B}_0$, and the source signals are estimated by $\boldsymbol{s}(t) = \boldsymbol{A}^+\boldsymbol{x}(t) = \boldsymbol{B}_0^{-1}\boldsymbol{z}(t)$.

4 Simulations

The performance index (also called the estimation error) in this Section is given in Refs. [5,9].

Example 1. An array of 6 sensors receives the three independent source signals. The mixture matrix is $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \boldsymbol{a}(\theta_3)]$, where $\boldsymbol{a}(\theta) = [1, \exp(-\mathrm{j}\pi\cos(\theta)), \cdots, \exp(-\mathrm{j}5\pi\cos(\theta))]^{\mathrm{T}}$ denotes the response vector of a six-element uniform linear array with half wavelength sensor spacing. The three deterministic independent sources are defined by $s_1(t) = \sqrt{2}\sin(600t + 5\cos(60))$, $s_2(t) = 2\sin(350) \times \sin(30t)$ and $s_3(t) = \sqrt{2}\sin(150t)$. The arrival directions of the three sources are 80°, 90°and 100°, respectively. Array receives the above three source signals together with stationary complex spatially colored noises, and SNR is 20 dB. The sampled number is 1000. The source signals shown in Fig. 1 are estimated by jointly approximate diagonalization of the five eigen matrices.

Example 2. In order to test the computational efficiency of the proposed method, the mixture matrix $\boldsymbol{A}$ = randn(151, 25) + j* randn(151, 25) was

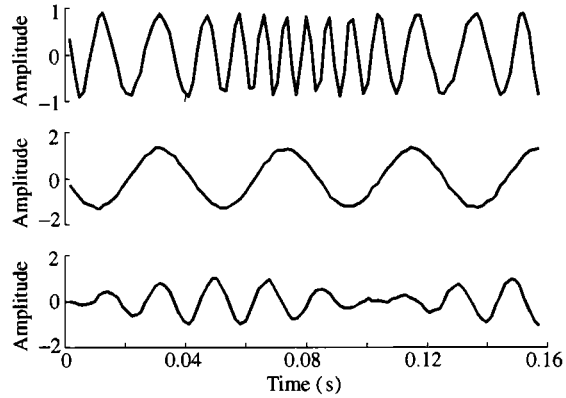

Fig. 1. The three estimated source signals of $s_1(t)$, $s_3(t)$ and $s_2(t)$, respectively, from top to bottom.

randomly generated and was kept unchanging in the independent trials. Five eigen matrices were generated randomly in the twenty independent trials by the following equations:

$$\boldsymbol{R}_x(0) = \boldsymbol{A}\,\mathrm{abs}[\,\mathrm{diag}(\mathrm{randn}(25,1) + \mathrm{j}^{*}\,\mathrm{randn}(25,1))]\boldsymbol{A}^{\mathrm{H}} + \mathrm{diag}(\mathrm{randn}(151,1)),$$

$$\boldsymbol{R}_x(\tau) = \boldsymbol{A}\,\mathrm{diag}(\mathrm{randn}(25,1) + \mathrm{j}^{*}\,\mathrm{randn}(25,1))\boldsymbol{A}^{\mathrm{H}}.$$

The estimation error of the array response is shown in Fig. 2, and the computational time added by Matlab software is given in Fig. 3. It shows that the proposed method improves the estimated performance in the presence of the spatially colored noises. In the case with more than 25 sources, its computational efficiency is obviously better than that of the JADE. Here, the iteration algorithm converges to a fixed point for 9.64 rounds of iteration on average.

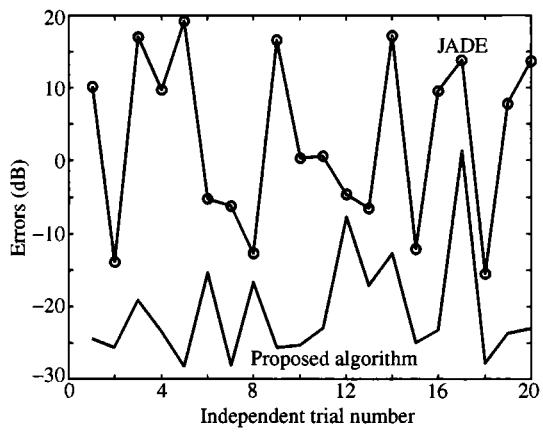

Fig. 2. Estimated Error of two methods versus independent trial number.

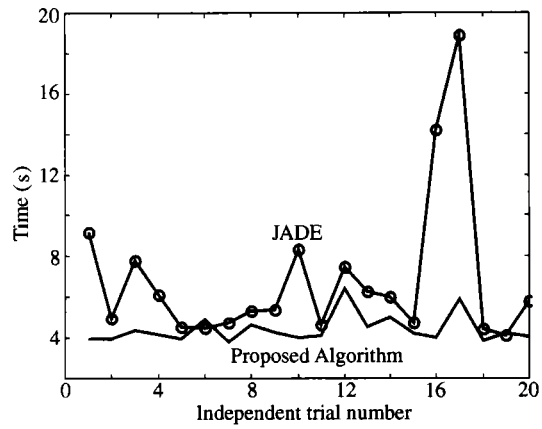

Fig. 3. Computational time of two methods versus independent trial number.

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